Evaluation of Free-Molecular Flow Passage Conductances with Thermal Radiation Analysis Software

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This paper describes a methodology for accurately evaluating the conductance of flow passages of virtually any geometry in the free-molecular flow regime. The approach utilizes commonly available thermal analysis software (a thermal radiation interchange factor code such as NEVADA/RENO[3]) thereby avoiding the costly development of specialized software for conductance calculations. Since this software is generally highly developed to include graphics capabilities and large libraries of available surface geometries, complex flow passages can be readily modeled. Thus, a major source of uncertainty in the analysis of rarefied gas flows—the evaluation of flow passage conductances—can be eliminated. The validity of this approach is demonstrated by agreement between calculated and experimentally measured conductance values for a variety of different flow passage geometries. Finally, flow passages from the Space Telescope venting analysis are utilized to demonstrate several modeling techniques which may be employed to expedite conductance calculations.

Nomenclature

$A1$ = passage cross sectional area at entrance, 1.0/cm^2
Aij = "Script F" factor = fraction of the (black body) radiant energy leaving surface i (entrance) which is deposited on surface j (exit), including transport by reflection
Bij = "transport probability" = the probability that a molecule entering the flow passage entrance (surface i) will reach the exit (surface j)
C = flow passage conductance, l/s
Gij = "Gebhart Factor" = fraction of the radiant energy leaving surface i (entrance) which is deposited on surface j (exit), including transport by reflection
M = molecular weight of gas, g/mole
$\dot{M}$ = flow passage throughput (flow rate), Torr l/s
P = flow passage pressure differential, Torr
T = gas temperature, K

Introduction

IN the design and development of aerospace systems, it is often necessary to analyze the flow of gas in the free-molecular flow regime through systems of complex geometry. The free-molecular (collisionless) flow regime is defined by the requirement that the mean free path between intermolecular collisions be greater than the smallest characteristic dimension of the system. Under these conditions, the most common analytical approach is to divide the system volume into subvolumes (chambers) and interconnecting flow passages. These subvolumes must be selected so that the molecular number density is approximately uniform and the velocity distribution function is approximately isotropic within the subvolume. In modeling the system for analysis, the subvolumes are replaced by lumped parameter nodes, while the interconnecting passageways are represented as conductances. When mass conservation is imposed on the system model, a first-order, ordinary, coupled system of differential equations is obtained.

Before the model behavior can be determined from the solution of this system of equations, the conductances of the flow passageways must be evaluated. The most common approach to the evaluation of vacuum flow conductances is to attempt to model the actual passage geometry with one or more of the limited number of available solutions. These solutions tend to be for elementary geometries such as cylindrical and rectangular ducts.[1,2] Naturally, for flow passages of complex geometry, such approximate methods may yield conductance values which are substantially in error.

This paper describes a method for accurately evaluating the conductance of flow passages of virtually any geometry utilizing commonly available software (a thermal radiation interchange factor code such as NEVADA/RENO[3]). Thus a major source of uncertainty in analyzing rarefied gas flows can be eliminated without the substantial investment required to develop specialized software for conductance calculations. In order to validate this approach, calculated and experimentally measured conductance values for a wide variety of flow passage geometries are compared. The results of these comparisons substantiate the capability of this methodology to accurately evaluate flow passage conductances. To demonstrate the application of this approach to actual spacecraft flow passages and illustrate several useful modeling techniques, three conductance calculations from the Space Telescope[4] venting analysis[5] are reviewed.

Conductance Evaluation by Monte Carlo Simulation

Since experimental evaluations tend to be time consuming, costly, and of limited accuracy, flow passage conductances in the free-molecular regime are evaluated analytically. One analytical approach involves applying mass conservation at each differential surface area element of the flow passage. This yields an integral equation, which can be solved in closed form for only the simplest passage geometries.[6]

An alternative approach utilizes direct Monte Carlo simulation to evaluate the probability that a molecule entering the flow passage will exit the opposite end (transmission probability B). This is accomplished by following the paths of a large number of particles through the flow passage until they reach the exit or are reflected back through the entrance. Since particle number densities and distribution functions are assumed to be uniform within each of the subvolumes connected by the flow passage, particle trajectories are initiated uniformly across the entrance planes with a "cosine" distribu-

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tion (Lambert's law). After particles leave the entrance plane, they are reflected from one flow passage surface to another until they reach either the exit or entrance surfaces.[7] Since particle reflections are assumed to be diffuse in nature, the direction of a reflected particle's trajectory is determined randomly with a cosine distribution. This assumption is supported by experimental results for engineering surfaces at near normal temperatures.[6] However, any degree of specular reflectivity can easily be included in the simulation process. After a sufficient number of particles have been followed, the transport probability can then be evaluated as the ratio of the number of particles reaching the exit to the total number initiated at the entrance. An estimator for the variance of the calculated transport probability can be obtained using standard statistical analysis techniques. With the transmission probability determined for a given flow passage B_{12}, the conductance may be directly calculated from the following relationship.[8]

$$C_{12} = 3.64 A_1 \left(\frac{T}{M}\right)^{1/2} B_{12} \qquad (1)$$

While in principle the transport probability can be evaluated to any desired degree of accuracy if enough particles are followed, computer costs will ultimately limit that accuracy. However, experience has shown that for most passage geometries, the transport probability can be determined to a reasonable accuracy level (90% confidence level $\pm 10\%$ of B) with relatively modest expenditures of computer time (<600 s on UNIVAC 1100).

Conductances from Radiation Exchange Factors

The cost of developing simulation software for the wide variety of complex surface geometries encountered in actual flow passageways can be quite substantial, and may be the barrier that has retarded the application of direct simulation techniques first developed in the early 1960s.[7,9] These software development costs can be eliminated by applying existing radiation exchange factor codes. This approach is based on the analogy between free-molecular flow and the transport of short wavelength (ray model–no diffraction) electromagnetic radiation.

The behavior of rays of electromagnetic radiation and gas molecules under free-molecular conditions are identical if the exit and entrance surfaces are totally absorbing (black) and the internal passageway surfaces are nonabsorbing diffuse reflectors. Under these conditions, the radiation exchange factor [Gebhart Factor (Gij)[10] or "Script F" Factor (Aij)[11]] between the entrane and exit surfaces is identical to the transport probability.

$$Gij = Aij = Bij$$

Thus, Monte Carlo-based codes developed to evaluate radiation exchange factors can be directly applied to calculate transport probabilities from which the flow passage conductance values may be readily obtained.

Alternately, the radiation exchange factors Gij or Aij can be evaluated from the geometric configuration (view) factors. Software packages to perform this calculation are contained in most large thermal analyzer codes.[11] As in radiation heat transfer analyses, care must be taken to insure that the discretization of passageway surfaces is fine enough to adequately represent the spatial variation of incident fluxes. This is not a consideration with Monte Carlo-based simulation codes, which take into account surface flux variations.

All of the conductance calculations described in this report were performed with the RENO/NEVADA[3] Monte Carlo-based simulation software package. Many of the available radiation exchange factor codes are highly developed, incorporating extensive libraries of different surface geometries and graphics capabilities for viewing model geometry. These features allow flow passages of complex geometry to be readily modeled, as exemplified by Figs. 1 and 2.

Methodology Validation

In order to demonstrate the validity of the conductance evaluation methodology described in this paper, comparisons were made between calculated and experimentally measured values for a variety of flow passage geometries. In Figs. 3 and 4, transmission probabilities calculated using the procedure described in this report [marked with 90% confidence interval symbols (I)] are compard with experimental measured values (o)[9] for eight flow passage geometries. In Fig. 5, computed and measured[9] conductance values are tabulated for "bulged elbow" flow passages with three geometries. From Figs. 3, 4, and 5, it may be seen that the calculated values are reasonably close to the experimentally measured values for all of the considered flow passage geometries.

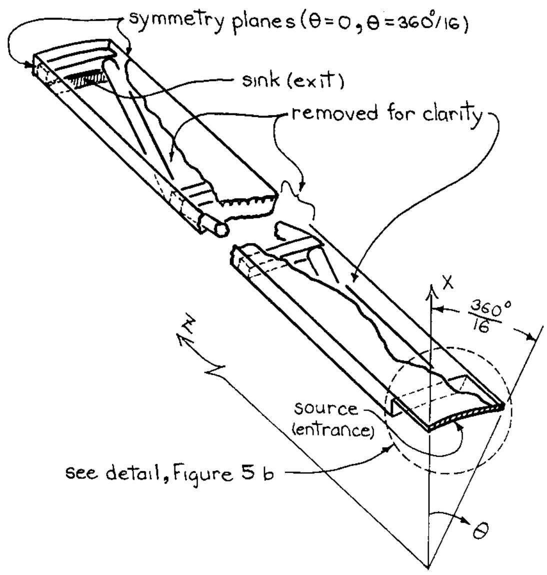


Fig. 1 Utilization of symmetry planes greatly simplifies modeling of a complex Space Telescope flow passage.

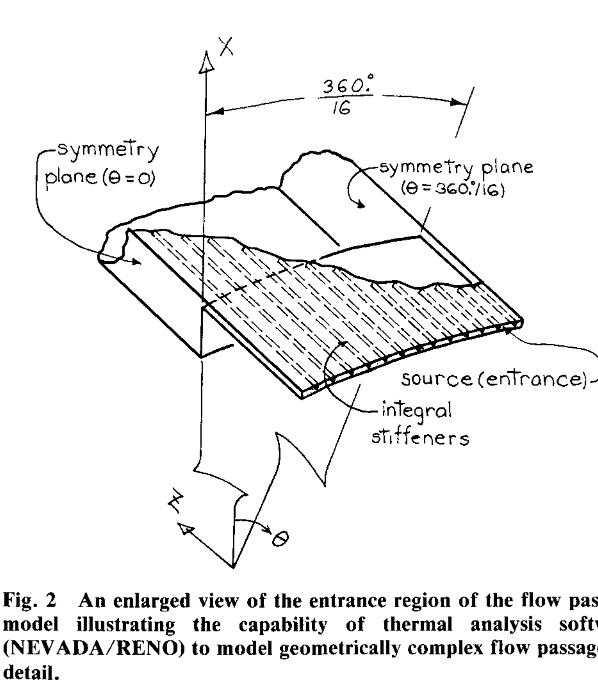


Fig. 2 An enlarged view of the entrance region of the flow passage model illustrating the capability of thermal analysis software (NEVADA/RENO) to model geometrically complex flow passages in detail.

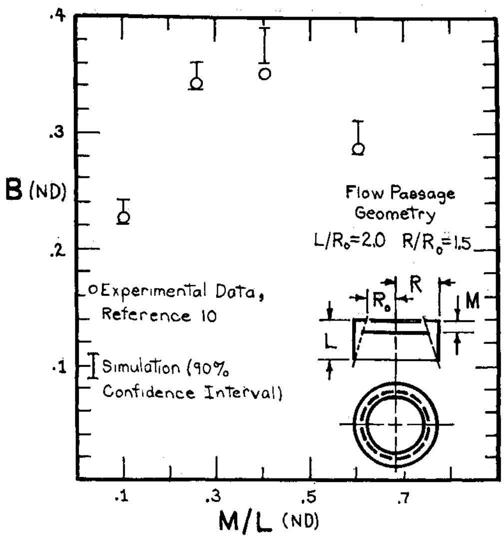


Fig. 3 Comparison of calculated and experimentally measured transmission probabilities (B) for a cylindrical flow passage with one restricted end and a disk-shaped baffle plate.

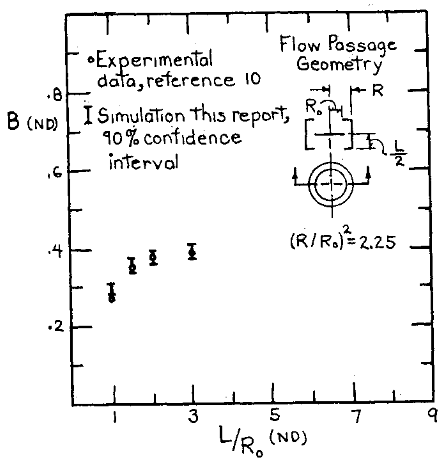
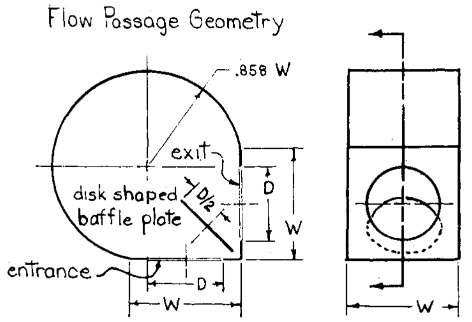


Fig. 4 Comparison of calculated and experimentally measured transmission probabilities (B) for a cylindrical flow passage with two restricted ends and a disk-shaped baffle plate.



W/D	B exp.*	B simulation, 90% confidence
1.00	.32	.284 ≤ .297 ≤ .310
1.33	.39	.382 ≤ .397 ≤ .412
2.00	.44	.440 ≤ .466 ≤ .492

* experimental data, reference 10

Fig. 5 Comparison of calculated and experimentally measured transmission probabilities (B) for a "bulged elbow" flow passage incorporating a disk-shaped baffle plate.

Since this simulation procedure was a key element of the Space Telescope venting analysis, it was deemed necessary to demonstrate its validity on a representative spacecraft flow passage. The light baffle located within the Space Telescope aft bulkhead vent was selected (Fig. 6). The experiment consisted of flowing dry nitrogen gas from a source of known flow rate into a "randomizing" chamber which was directly connected to a model of the flow passage to be measured (Fig. 7). After flowing out of the chamber and through the flow passage, the dry nitrogen gas exited into a vacuum chamber. The pressure drop across the flow passage was calculated as the difference between the measured pressure values in the randomizing and vacuum chambers. The flow passage conductance C was calculated as the ratio of the mass flow rate $\dot{M}$ through the passageway to the pressure drop Δp across it (Fig. 7).

$$C = \dot{M}/\Delta p$$

A more detailed discussion of this experiment is contained in Ref. 5. The experimentally measured and simulation calculated baffle conductance values are given in Table 1. The extremely close agreement between the measured and calculated values is a fortuitous coincidence because the claimed calibration accuracy for the ion gages is ±13.5%.

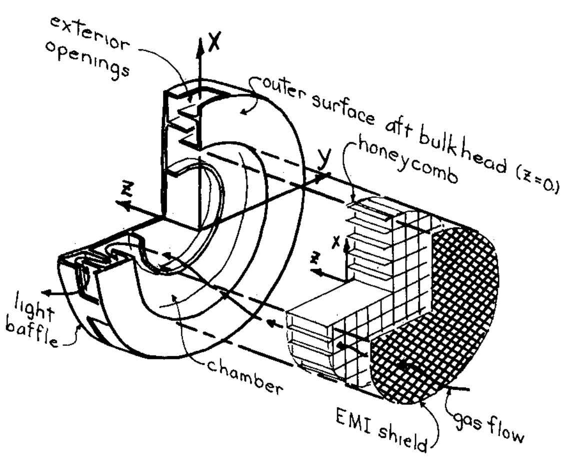


Fig. 6 Aft bulkhead vent assembly, Space Telescope.

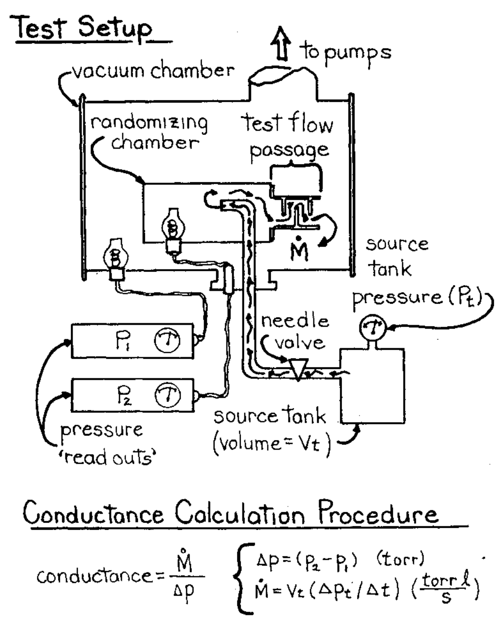


Fig. 7 Experimental setup and data reduction procedure for flow passage conductance measurements.

However, both this and the preceding comparisons demonstrate that the conductance evaluation procedure described in this report can accurately predict conductance values for a wide variety of flow passage geometries.

Applications

In the modeling of free-molecular flow passages there are several techniques which can often be employed to facilitate conductance calculations.

One technique to simplify the analysis of some flow passages is initiated by dividing them into separate subpassages. The conductance of each subpassage is then determined through simulation or analytical calculation, and the conductances are combined to obtain the complete passageway conductance. Use of this technique is only appropriate when the subpassages are connected by chambers of sufficient size to randomize the flow (uniform, isotropic velocity distribution function) over the subpassage entrance-exit surfaces. If this condition is met, however, this technique can greatly facilitate the analysis of flow passages with low transmission probabilities (~5%) or conductances. The difficulty of flowing molecules through this type of passageway may result in the expenditure of large amounts of computer time to evaluate the conductance with sufficient accuracy. Division of the passageway into subpassages with large conductances will often reduce computer time requirements. Also, it may be possible to save additional computer time by evaluating some of the subpassage conductances from known analytical solutions.

Application of this technique is appropriate to the analysis of the Space Telescope aft bulkhead vent (Fig. 6), since its two main elements (the baffle and honeycomb) are connected by a chamber which should substantially randomize the flow. The regular hexagonal configuration of the honeycomb subpassage allows its conductance to be evaluated with reasonable accuracy from the analytical solution for a cylindrical tube.[2] The conductance of the geometrically complex baffle, however, requires a simulation calculation. The total conductance of the vent is calculated by combining the two subpassage conductances and the EMI shield in series (Table 2). Although some degree of engineering judgment is required, this technique can often be employed to expedite conductance calculations.

Another technique is available to simplify conductance calculations for geometrically symmetrical flow passages. Specular, perfectly reflecting (no absorption) surfaces are utilized to simulate symmetry planes in the computational model (Fig. 8). This simulation technique can be supported by

Table 1 Baffle conductance values: simulation calculated and experimentally measured (see Fig. 7)

Simulation calculated value = 202 ± 19 l/s ($B = 0.0606 \pm 9.5\%$)

	Measured values			
Test no.	C, l/s	M, t l/s	P_2, T	P_1, T
3	209	0.159	$1.2\text{-}10^{-3}$	$4.4\text{-}10^{-4}$
4	210	0.052	$3.2\text{-}10^{-4}$	$7.1\text{-}10^{-5}$

Table 2 Calculated conductance aft bulkhead vent

Subpassage conductances (see Fig. 6)
EMI shield: $C_e = 4470$ l/s ($B_e = 0.550$)
Honeycomb: $C_h = 1121$ l/s ($B_h = 0.137$)
Baffle: $C_b = 594 \pm 5.9\%$, l/s ($B_b = 0.0724$)
Total conductance, one vent assembly
$(1/C_T)^{-1} = (1/C_e + 1/C_h + 1/C_b)^{-1} = 357$ l/s ($B_T = 0.0436$)

the following heuristic argument: as a computed molecular trajectory undergoes specular reflections (angle of incidence equals angle of reflection) from the symmetry plane, its path can be interpreted as two molecular trajectories. These trajectories are constructed by 1) extending the pre-reflection and post-reflection computational trajectories across the symmetry plane and 2) assuming identical diffuse reflections from symmetric passage surfaces across the symmetry plane (broken lines) (Fig. 8). This system of path lines can be interpreted as the superposition of two molecular trajectories which are symmetrical about the symmetry plane (Fig. 9). These trajectories consistently model molecular interaction with passageway surfaces on both sides of the symmetry plane. Thus, the computational model consisting of passageway surfaces on only one side of the symmetry plane correctly simulates the entire symmetrical passageway geometry.

The validity of the above argument and its associated modeling technique can be readily tested by comparing simulation calculations for symmetrical flow passageways modeled with and without symmetry planes. In the flow passages which have been tested, good agreement has always been obtained. In Fig. 10, the transmission probabilities for the aft bulkhead vent baffle simulated with a one-eighth sym-

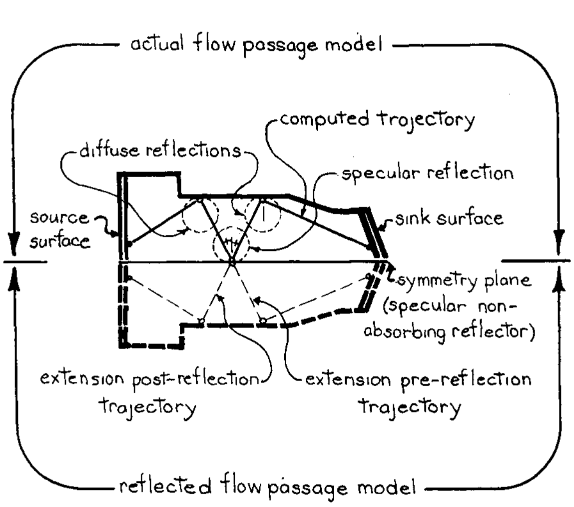


Fig. 8 Construction of a pair of trajectories from a single computational trajectory for a representative flow passage model incorporating a symmetry plane.

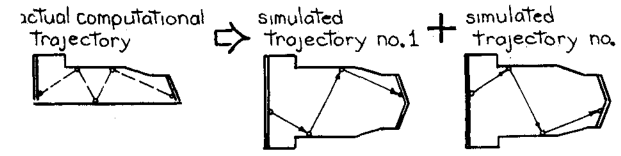


Fig. 9 Relationship of a computational trajectory and its associated trajectory pair in a representative flow passage model incorporating a symmetry plane.

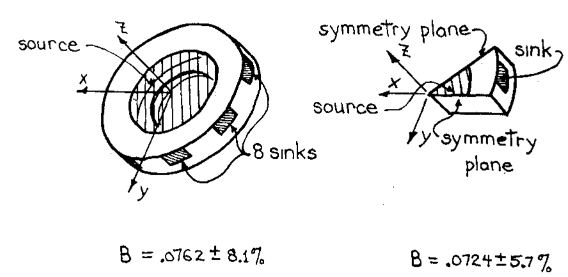
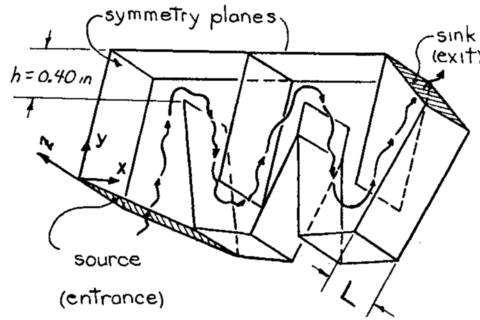


Fig. 10 Comparison of the calculated transmission probability values (B) for a one-eighth symmetry model (right) and a full model (left).



B ≡ transport probability (90% confidence intervals)
L ≡ seal length, z direction
Model a) $L = 1.00 \cdot 10^6$ in. ≈ >; B = .0342 ± 9.8%
Model b) L = 1.0 in. w/symmetry planes; B = .0348 ± 3.2%.

Fig. 11 Space Telescope WFPC seal demonstrating use of symmetry planes in modeling two-dimensional flows.

metry model are compared with a full model (no symmetry planes).

An excellent example of the utilization of symmetry planes to simplify the simulation of a complex flow passage is provided by the Space Telescope "mirror ring" vent (Figs. 1 and 2). This spatially complex flow passage consists of 16 symmetric cylindrical sectors. The model simulating the entire mirror ring vent consists of a single symmetrical sector (1/16) of the flow passage bounded by two symmetry planes (specular, nonabsorbing surfaces). Clearly, symmetry planes can greatly expedite the modeling of flow passages with a high degree of geometrical symmetry.

In addition to simplifying the simulation of complex symmetrical passages, symmetry surfaces can be utilized to model passages with two-dimensional flows. The Space Telescope WFPC seal provides an example of such a flow passage (Fig. 11). Its prismatic Z-axis cross section and quasi-infinite length along the Z axis ($L/h = 508 \gg 1$) render the flow independent of the Z coordinate. This two-dimensional (x-y plane) flow may be simulated by considering a unit length of seal bounded by two specular, nonabsorbing x-y plane surfaces. The validity of this model is demonstrated by the agreement between transmission probabilities calculated for a duct model with quasi-infinite length (Fig. 11, model a) and the unit length model bounded by symmetry planes (Fig. 11, model b). The conductance for the complete seal is obtained from the product of the seal length and the conductance calculated for the unit length model.

Conclusions

The methodologies described in this paper can accurately evaluate the free-molecular conductance of flow passages of virtually any geometry. The validity of this approach has been demonstrated by comparison with experimentally measured conductance values for a wide variety of flow passage geometries. By employing widely available thermal radiation analysis software, this approach avoids the substantial costs associated with developing specialized software for conductance calculations. The large libraries of surface geometries available in many thermal radiation analysis codes allow complex flow passage geometries to be quickly and economically analyzed. Thus, the substantial uncertainty in conductance values evaluated with approximate methods can be reduced, and the accuracy of free-molecular flow analyses improved.

References

[1]Santeler, D.J., Holkeboer, D.H., Jones, D.W., and Pagano, F., "Vacuum Technology and Space Simulation," NASA SP-105, 1966, Chap. 5.

[2]Dushman, S., *Scientific Foundations of Vacuum Technique*, 2d ed., John Wiley, New York, 1962, Chapter 2.

[3]*NEVADA/RENO Software Package,* Turner Associates, P.O. Box 426, Brea, CA.

[4]Bahcall, J.N. and Spitzer, L., "The Space Telescope." *Scientific American*, Vol. 247, July 1982, pp. 40–51.

[5]Peterson, M.A. and Lee, A.L., "ST Venting Analysis Update," Lockheed Missiles and Space Company, Sunnyvale, CA, TSS232, January 1984.

[6]Bird, G.A., *Molecular Gas Dynamics*, Clarendon Press, Oxford, 1976, p. 100.

[7]Davis, D.H., "A Monte Carlo Calculation of Molecular Flow Rates Through a Cylindrical Elbow and Pipes of Other Shapes," *Journal of Applied Physics,* Vol. 31, July 1960, pp. 1169–1176.

[8]Holland, L. and Jones, A.S., *Vacuum Manual,* E. & F. Spon Co., London, 1974, p. 21.

[9]Levenson, L. L., Milleron, N., and David, D.H., "Optimization of Molecular Flow Conductance," *Transactions of the Seventh National Symposium on Technology,* edited by C.R. Meissner, Pergamon Press, New York, 1961, pp. 372–377.

[10]Siegel, R. and Howell, J.R., *Thermal Radiation Heat Transfer*, McGraw-Hill, New York, 1972, Appendix E, pp. 798–800.

[11]"Thermal Network Modeling Handbook," NASA CR-144418, Jan. 1972, pp. 38 and 50.